# The relation between nuclear charge radii and other parameters characterizing the nuclear drops.

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Correlation between the rms nucleus charge radius and their deformation, isospin asymmetry, mass and charge numbers are presented. Four parameters radius parametrization formula is proposed. Ratio of experimental to theoretical value distribution exhibit variation equal to 0.0044, i.e. it corresponds to the value of experimental errors. Observed correlations can impose interesting constraints on the form of the nuclear equation of state.

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In the laboratory framework the nuclear matter is available only as the atomic nuclei, which can be viewed as the liquid drops of nuclear matter. Because the dimension and shape of nuclei reflect complicated interactions inside nuclear matter it is of great importance to find correlations between the parameters describing the nuclear drops. In this letter we demonstrate new simple relation between these parameters, based on extensive data for 574 heavy nuclei. In particular, we choose the relation between nuclear charge radii and their deformation, isospin $I$, mass $A$ and charge numbers $Z$. In order to describe the nucleus deformation one can introduce different kinds of the shape parametrization [1]. In our approach description of deformations is limited to axially symmetric quadruply deformed shapes and therefore for every nuclei the respective $\beta$ parameters are attributed. Thess $\beta$ parameters are assumed to be taken from model calculations presented in [1].

The reduced nuclear charged radius can be defined as

$$r(Z,I,\beta) = \frac{\left\langle R^2\left(\beta Z, I\right)\right\rangle^{1/2}}{A^{1/3}} \qquad (1)$$

where $\left\langle R^2\left(\beta Z, I\right)\right\rangle^{1/2}$ is the root mean square (hereafter rms) nuclear charge radius. The distribution of these reduced radii (normalized by a constant $r_0$, as explained in the further part), based on experimental data [2] for $N = 574$ nuclei with $A$>100 is presented in fig. 1 as a dashed line and exhibits variations about of 3%.
Correlations between radius $r$ and $\beta$,$Z$, $A$ and $I$ parameters are presented on fig. (2).

Looking at this figure, we cannot see clearly any systematical behavior. This fact is related to confounded dependence on all parameters. Main result of this letter is the procedure allowing us to unmask this dependence and unravel generic correlations.
Since deviations in the rms distribution are small, it is natural to adopt the form of an expansion, using simple parametric form

$$r = r_0\left(1+\alpha_\beta\cdot\beta^2\right)\left(1+\alpha_Z\cdot(Z-50)\right)\left(1+\alpha_I\cdot I\right) \qquad (2)$$

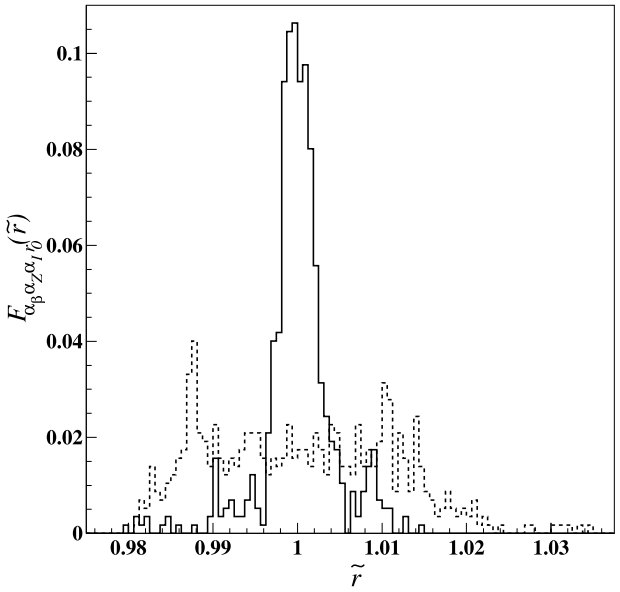


Figure 1: The distribution $F_{\alpha_\beta\alpha_Z\alpha_I r_0}(\tilde{r})$. The solid lines represents the distribution of $\tilde{r}$ for parameters $\alpha_\beta$, $\alpha_Z$ ,$\alpha_I$, $r_0$ given by (4). Dashed line represents this some distribution for parameters $\alpha_\beta = 0$, $\alpha_Z = 0$, $\alpha_I = 0$ and $r_0 = 0.945$ what for such case is the distribution of $r^{exp}/r_0$.

in which $\alpha_\beta$, $\alpha_Z$, $\alpha_I$ and $r_0$ are free coefficients. The dependence on $\beta^2$ results from the dependence of rms radius on $\beta^2$ for quadruply deformed shape. The form of the dependence on $I$ is justified by the results obtained in the recent papers [3] where an important observation was made, that for heavy nuclei, their nuclear rms charge radii exhibit a systematic variation as a function of neutron-proton asymmetry.

We can find coefficients $\alpha_\beta$, $\alpha_Z$, $\alpha_I$ and $r_0$ by consid-

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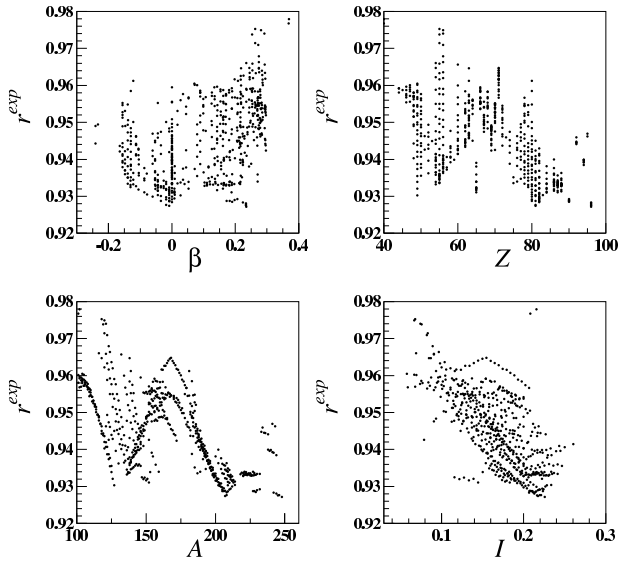


Figure 2: Correlation between reduced radii and respectively $\beta$, $Z$, $A$ and $I$.

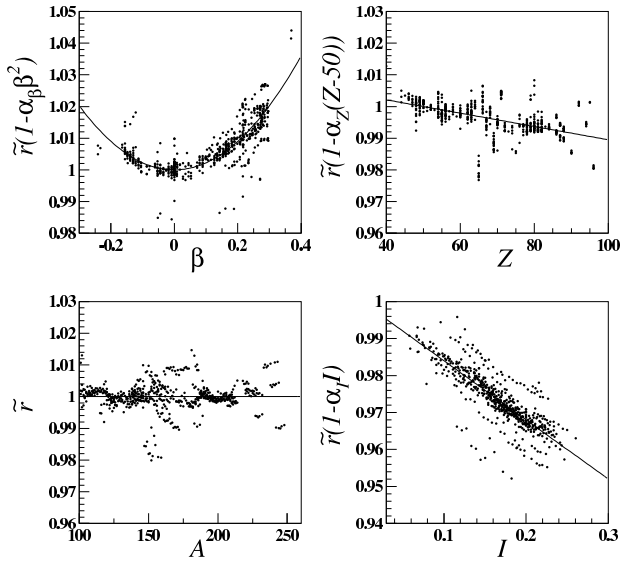


Figure 3: Correlation between reduced radii with "switch-off" dependence from value other than correlated one, and correlated values are respectively $\beta$, $Z$, $A$ and $I$. Solid lines represents for consider correlation fit given by (2)

ering distribution of value $\tilde{r}$ defined as

$$\tilde{r} = \frac{r^{exp}}{r_0\,(1+\alpha_\beta\cdot\beta^2)\,(1+\alpha_Z\cdot(Z-50))\,(1+\alpha_I\cdot I)} \quad (3)$$

where the $r^{exp}$ is the experimental value of the reduced nuclear charged radius. Now the distribution $F_{\alpha_\beta\alpha_Z\alpha_I r_0}(\tilde{r})$ can be treated as a function of $\alpha_\beta$, $\alpha_Z$, $\alpha_I$ and $r_0$. If the parametrization (2) is correct, the distribution of $\tilde{r}$ should tend to the Dirac delta function $\delta(\tilde{r}-1)$ for some specific values of $\alpha_\beta$, $\alpha_Z$, $\alpha_I$ and $r_0$. By minimizing the variance of the distribution $F_{\alpha_\beta\alpha_Z\alpha_I r_0}(\tilde{r})$ under the condition that its average value is equal to 1 the following set of parameters was obtained:

$$\alpha_\beta = 0.225\ \ \alpha_Z = -0.00021\ \ \alpha_I = 0.16\ \ r_0 = 0.9681\,\mathrm{fm} \quad (4)$$

To verify the equation (2), we plot on fig. 2 respective correlation and curves obtained from fit (2). We notice the systematic and consistent behavior for almost all charge radii, confirming the soundness of chosen parametrization.

As we can see, for the majority of considered nuclei the chosen form of parametrization works quite well. However, there are still some groups of points on the plot which do not match the major trends.

To see the structure of these particular data better, for correlation $\tilde{r}\,(1+\alpha_I\cdot I)$ *vs.* $I$ we separate this group by dashed lines given respectively by equation $0.988+\alpha_I\cdot I$ , $0.995+\alpha_I\cdot I$ and $1.005+\alpha_I\cdot I$. The points located on the main branch are marked by green color whereas the points above line $1.005+\alpha_I\cdot I$ and between lines $0.988+\alpha_I\cdot I$ and $0.995+\alpha_I\cdot I$ are marked by colors blue and yellow, respectively. Eleven terbium points which are located below the line $0.988+\alpha_I\cdot I$ were marked separately by red color. As one can see green points match perfectly all correlations announced by us. Other points follow also similar trends, but he corresponding curves are slightly shifted comparing to the trends exhibited by green points. Two reasons can be responsible for such behavior. First, the systematics of the discrepancies can be associated with experimental and systematical errors. Second, the effect may be the consequence of some special properties of these nuclei. We should point out that radii of terbium isotops (red points) are measured within total (experimental + systematical) error about 3% what is quite a large uncertainty whereas the relative error of measured terbium radii with respect to the radius of the reference isotope is much smaller and reads 0.04% [2]. It is therefore possible that that observed radii deviations can origin from the total experimental error.

We summarize our conclusions:

1. Correlation observed by us proves that the functional form of radius parametrization is correct for majority of consider nuclei. Ratio of experimental

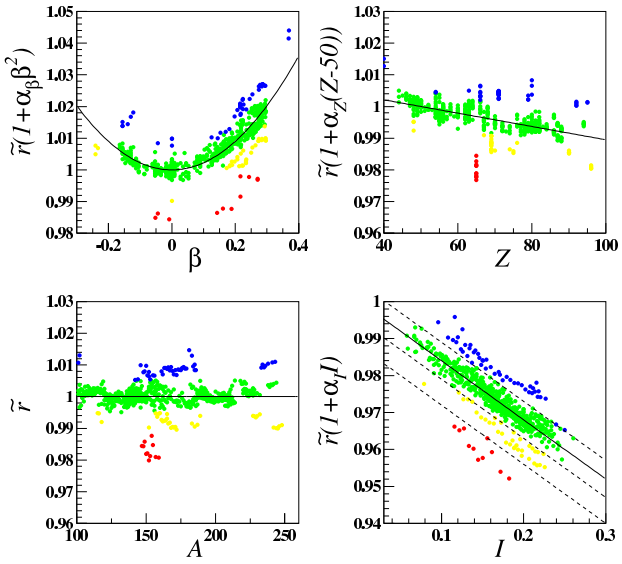


Figure 4: Further analysis of correlations as presented in Fig 3. Color of the points is determined by positions on correlations between the isospin $I$ and $\tilde{r}\,(1+\alpha_I \cdot I)$. The points from the main branch are marked by green color and are located between dashed lines with respect to the equation $0.995+\alpha_I \cdot I$ and $1.005+\alpha_I \cdot I$. Points located above line $1.005+\alpha_I \cdot I$ are plotted in blue color whereas points below line $0.995+\alpha_I \cdot I$ and above line $0.988+\alpha_I \cdot I$ are plotted in yellow color. Eleven points, located below line $0.988+\alpha_I \cdot I$ corresponding to the terbium isotopes are plotted in red color.

to theoretical value for radii is presented on fig. 1 as a solid line. The variation of this distribution is equal to 0.0044, i.e. it corresponds to the value of experimental errors.

2. The correlation between radius and the coefficient of deformation clearly fits a parabolic shape and shows that in nuclear ground state the dominating deformation is the quadrupole one. However it should be stresses that value of $\alpha_\beta = 0.225$ is more than two times higher from the value expected for uniform quadruply deformed matter. This is an interesting result which may point at strong correlations between non-uniformity of the density of protons and quadrupole deformation. Correlation between radii and isospin asymmetry is in agreement with the earlier observations [submitted to [3] .

3. Effects of the surface can be easily taken into account, by adding another functional parametric dependence with additional parameter multiplying the $A^{-1/3}$ term. We have found that these effects are negligible. This observation is consistent with our choice of considering only heavy nuclei, where surface effects are not expected to play the major role.

4. Correlation between radii $r$ and $Z$ shows, that at value of deformation and neutron-proton asymmetry fixed, protons become on average closer, as the number of protons increases. This observation probably suggests the possibility of the creation of the neutron skin. This correlation accompanied by the correlation between radii and the isospin can impose interesting constraints on the form of the nuclear equation of state. This possibility would be addressed in the forthcoming publication.

5. Taken into account the systematics of deviations for the terbium isotopes, it would be interesting to obtained and compare experimental data corresponding to smaller errors. In the future the shift from systematical behavior also for isotopes represented by blue and yellow points should be considered and explained.

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